University of Cambridge

Course No.	Title	Instructor	Grade	Textbook	Subject Matter
NA	Elliptic Partial Differential Equations	N. Wickramasekera & Dr. G. Taujanskas	NA	1. D. Gilbarg and N. Trudinger, Elliptic partial differential equations of second order. 2. L. Simon, Schauder estimates by scaling. Calc. Var. & PDE, 5, (1997), 391–407.	The course will provide a rigorous treatment, based on a priori estimates, of both classical and weak solutions to linear elliptic equations, focusing on the question of existence and uniqueness of solutions to the Dirichlet problem and the question of regularity of solutions. Specific topics include: • harmonic functions • maximum principles for general second order equations • Schauder estimates (via L. Simon's scaling argument) • the continuity method for existence of solutions • solvability of the Dirichlet problem in balls • Perron's method • divergence form operators • De Giorgi–Nash–Moser estimates • the Harnack theory • as time permits, a brief discussion of the quasilinear theory centred around the prototypical example of the Minimal Surface Equation.

NA	Stochastic Calculus and Applications	Professor J. Miller	NA	1. R. Durrett Probability: theory and examples. Cambridge, 2010. 2. I. Karatzas and S. Shreve Brownian Motion and Stochastic Calculus. Springer, 1998. 3. P. Morters and Y. Peres Brownian Motion. Cambridge, 2010. 4. D. Revuz and M. Yor, Continuous martingales and Brownian motion. Springer, 1999. 5. L.C. Rogers and D. Williams Diffusions, Markov Processes, and Martingales. Cambridge, 2000.	This course will be an introduction to Itˆo calculus and will aim to cover the following topics. • Brownian motion. Existence and sample path properties. • Stochastic calculus for continuous processes. Martingales, local martingales, semi-martingales, quadratic variation and cross-variation, Itˆo's isometry, definition of the stochastic integral, Kunita-Watanabe theorem, and Itˆo's formula. • Applications to Brownian motion and martingales. L´evy characterization of Brownian mo- tion, Dubins-Schwartz theorem, martingale representation, Girsanov theorem, conformal invariance of planar Brownian motion, and Dirichlet problems. • Stochastic differential equations. Strong and weak solutions, notions of existence and uniqueness, Yamada-Watanabe theorem, strong Markov property, and relation to second order partial differential equations. • Stroock–Varadhan theory. Diffusions, martingale problems, equivalence with SDEs, ap-

					proximations of diffusions by Markov chains.
NA	Schramm-Loewner Evolutions	Dr. Y. Yuan	NA	1. Wendelin Werner, Random planar curves and Schramm-Loewner evolutions, 2004. Also available at https://arxiv.org/abs/math/0303354. 2. Gregory F. Lawler, Conformally Invariant Processes in the Plane, volume 114 of Mathe- matical Surveys and Monographs. American Mathematical Society, Providence, RI, 2005.	The main goal of this course is to define and study SLE. The following topics will be covered: • conformal maps: relations to planar Brownian motion, distortion estimates, and the Loewner differential equation, • the definition of SLE, basic properties and their geometry, • relation to the Gaussian free field.
NA	Distribution Theory and Applications	Dr. A. Ashton	NA	1. F.G. Friedlander & M.S. Joshi, Introduction to the Theory of Distributions, C.U.P, 1998. 2. M. J. Lighthill, Introduction to Fourier Analysis and Generalised Functions, C.U.P, 1958. 3. G.B. Folland, Introdution to Partial Differential Equations, Princeton Univ Pr, 1995.	The course will first cover the basic definitions for distributions and related spaces of test functions. Then we look at operations such as differentiation, translation, convolution and the Fourier transform. We will introduce the Sobolev spaces Hs(Rn) and Hs loc(X) and describe them in terms of Fourier transforms for tempered distributions. The material that follows will address questions such as

					• What does a generic distribution look like? • Why are solutions to Laplace's equation always infinitely differentiable? • Which functions are the Fourier transform of a distribution with compact support? i.e. structure theorems, elliptic regularity, Paley-Wiener-Schwartz. The final section of the course will be concerned with H¨ormander's theory of oscillatory integrals.
NA	Advanced Probability	Professor Perla Sousi	NA	Brownian Motion, by Peter Morters and Yuval Perez	The aim of the course is to introduce students to advanced topics in modern probability theory. The emphasis is on tools required in the rigorous analysis of stochastic processes, such as Brownian motion, and in applications where probability theory plays an important role.
NA	Functional Analysis	Dr. A. Zsak	NA	Functional Analysis by Walter Rudin and Murphy, Gerard J. C∗-Algebras and Operator Theory. Academic Press, Inc., 1990	This course cover many of the major theorems of abstract Functional Analysis. It is intended to provide a foundation for several areas of pure and applied mathematics. We will cover the following topics: • Hahn–Banach Theorems on the extension of linear functionals. Locally convex spaces.

					• Duals of the spaces Lp(μ) and C(K). The Radon–Nikodym Theorem and the Riesz Representation Theorem. • Weak and weak-* topologies. Theorems of Mazur, Goldstine, Banach–Alaoglu. Reflexivity and local reflexivity. • Hahn–Banach Theorems on separation of convex sets. Extreme points and the Krein– Milman theorem. Partial converse and the Banach–Stone Theorem. • Banach algebras, elementary spectral theory. Commutative Banach algebras and the Gelfand representation theorem. Holomorphic functional calculus. • Hilbert space operators, C∗-algebras. The Gelfand–Naimark theorem. Spectral theorem for commutative C∗-algebras. Spectral theorem and Borel functional calculus for normal operators. Some additional topics time permitting. For example, uniform convexity and smooth-

					ness, ultraproducts, the Fr´echet–Kolmogorov Theorem, weakly compact subsets of L1(μ), the Eberlein–ˇSmulian and the Krein–ˇSmulian theorems, the Gelfand–Naimark–Segal con- struction.
NA	Analysis of PDE	Dr. Z. Wyatt	NA	Partial Differential Equations by Lawrence C. Evans	The following concepts will be studied: • well-posedness • the Cauchy problem for general (non-linear) PDE • characteristics • Sobolev spaces • elliptic boundary value problems (solvability and regularity) • evolutionary problems (hyperbolic, parabolic and dispersive PDE)
NA	Approximation Theory	Professor A. Shadrin	NA	E. W. Cheney, Approximation theory, MgGrow-Hill, New-York, 1966	The course consists of three parts. • We start with the classical approximation by polynomials, which includes the Weierstrass theorem, positive linear operators, and direct and inverse theorems for trigonometric ap- proximation.

					• We move then the emphasis to univariate splines which are piecewise polynomial functions. Here we study representation through the B-spline basis, spline interpolation theory and norm-minimization property of splines via orthogonal spline projector. • Finally, we make a tour into wavelets which will cover the multiresolution analysis and Daubechies orthogonal wavelets with a compact support.

Imperial College London

Course No.	Title	Instructor	Grade	Textbook	Subject Matter
MATH71035	Analytic Methods in PDE	Dr. A. Chandra	95.28%	Partial Differential Equations by Fritz John and Partial Differential Equations Jeffrey Rauch	This module introduces some of the partial differential equations appearing in physics and geometry, as well as a number of classical techniques to study them analytically. Topics include — Review of ODE Theory (Picard's Theorem, Gronwall's inequality) — Theory of first order quasilinear PDE (Methods of Characteristics) — Cauchy-Kovalevskaya Theorem (with sketch of the proof) — Holmgren's uniqueness theorem (with proof via Cauchy-Kovalevskaya, examples) — Laplace's equation (fundamental solution, regularity of harmonic functions, maximum principle, Green's function for a ball)

					— General second order elliptic equations (Existence and Regularity Theory, Fredholm Alternative) — Discussion of Schroedinger and Heat Equation (Schwartz space, Fourier techniques) — Wave Equation (Energy estimate, domain of dependence, domain of influence, fundamental solution, solution via Fourier techniques, Duhamel's principle)
MATH60029	Functional Analysis	Dr. P.F. Rodriguez	99.79%	Rabindranath Sen, A First Course in Functional Analysis Theory and Applications	An indicative list of topics is: Metric Linear Spaces and basic examples of topological spaces with non-metrisable topology. Minkowski and Hoelder Inequality. Existence of Hamel basis (axiom of choice 1st time). Normed vector spaces & example of not normed Frechet space (Schwartz test functions). Banach spaces. Classical Banach Spaces: l_p, c, c_0, L_p(μ), C(Ω), C^(m)(Ω). Closed Subspaces, Completness, Separability and Compactness in Classical Spaces. Schauder Basis. Continuous linear maps.

					Banach contraction mapping principle and applications to integral equations (Frdholm+Volterra). Finite dimensional spaces. The Hilbert space (orthonormal basis). The Riesz-Fisher Theorem. The Hahn-Banach Theorem. (Banach Limit.) Dual spaces: Dual spaces of classical spaces. Reflexive Non-reflexive spaces. Baire Cathegory Theorem (axiom of choice again). Principle of Uniform Boundedness. (Application to Fourier Series). Open Mapping and Closed Graph Theorems. Compact operators. Hermitian operators and the Spectral Theorem. The module provides a general orientation in contemporary research problems in Mathematical Analysis includ- ing PDEs , Stochastic Analysis, Dynamical Systems and Quantum Mechanics.
MATH60028	Probability Theory	Professor I. Krasovsky	98.07%	Probability, A. N. Shiryaev	An indicative list of topics is: Probability spaces. Random variables: (Bernoulli, Rademacher, Gaussian variables with

					integration by parts formula). Probability Distributions. Basic probability inequalities: Jensen, Tshebychev. Tail of Distribution Estimates. Convergence in probability, in p-th moment, almost everywhere. 0-1 Law. Mutual Independence of Events/Random Variable and Vieta Formula. Product Probability Spaces. Conditional Expectations and Independence. Borel-Cantelli Lemmas. Weak and Strong Laws of Large Numbers for Random Sequences and Series of Mutually Independent or Weakly Correlated Random Variables. Applications : [Probabilistic proof of Weierstrass Theorem, Monte Carlo Method for Large Dimensional Integration, Macmillan’s Theorem, Infinitely Often Events: Decay and Recurrence of Human Civilisations, Normal Numbers...] Weak Convergence & Characteristic Functions. Central Limit Theorem. Infinite Product of Bernoulli measures versus Gaussian measure.

MATH70054	Introduction to Stochastic Differential Equations and Diffusion Processes	Professor G.A. Pavliotis	86.89%	Stochastic Processes and Applications Diffusion Processes, the Fokker-Planck and Langevin Equations, Grigorios A. Pavliotis	The module is composed of the following sections: I - Introduction II - Elements of probability theory and of stochastic processes in continuous time III - Brownian motion and stochastic calculus IV - Stochastic integrals V - Stochastic differential equations VI - Applications to partial differential equations VII - Markov processes and invariant measures
MATH60017	Tensor Calculus and General Relativity	Dr. C. Ford	92.01%	'A First Course in General Relativity' (2nd Edition) by Bernard Schutz (Cam- bridge University Press).	This module will cover the following topics: 1. Special Relativity 2. Tensors in Special Relativity 3. Tensors in General Coordinates Systems 4. Parallel Transport and Curvature 5. General Relativity 6. The Schwarzschild Spacetime 7. Variational Methods

MATH60005	Optimisation	Dr. D. Kalise	95.18%	Introduction to Nonlinear Optimization, by Amir Beck	1. Mathematical preliminaries 2. Unconstrained optimization 3. Gradient descent methods 4. Linear and non-linear least squares problems 5. Stochastic gradient descent 6. Nature-inspired optimization 7. Convex sets and functions 8. Convex optimization problems and stationarity 9. KKT conditions 10. Duality 11. Introduction to dynamic optimization and optimal control.
MATH60026	Methods for Data Science	Dr. B. Bravi Dr. P. Thomas	94.2%	The Elements of Statistical Learning Data Mining, Inference, and Prediction, Second Edition, Trevor Hastie	- Introduction to computational tools for data analysis and visualisation; - Introduction to exploratory data analysis; - Mathematical challenges in learning from data: optimisation; - Methods in Machine Learning: supervised and unsupervised; neural networks and deepl earning; graph-based data learning;

					- Machine learning in practice: application of commonly used methods to data science problems; Methods include: regressions, k-nearest neighbours, random forests, support vector machines, neural networks, principal component analysis, k-means, spectral clustering, manifold learning, network statistics, community detection.
MATH60030	Fourier Analysis and the Theory of Distributions	Professor I. Krasovsky	95.18%	A Guide to Distribution Theory and Fourier Transforms, by R.S. Strichartz	The module will assume familiarity with measure theory and functional analysis, especially L^p spaces and linear functionals. Indicative content: Orthogonal systems in infinite-dimensional Euclidean spaces, Bessel inequality, Parseval equality, general Fourier series, trigonometric basis in L_2[-Pi,Pi], convergence of trigonometric Fourier series, Fejer's theorem and applications, Fourier transform and its properties, application to solution of differential equations, Plancherel theorem, Laplace transform, linear functionals, distributions, basic properties of distributions and applications, Fourier transform for distributions.

MATH50008	Partial Differential Equations in Action	Dr. T. Bertrand	94.61%	"Partial Differential Equations: An introduction", W. Strauss (Wiley)	The module is composed of the following sections: 1) Introduction: models in applied mathematics and basic properties of PDEs; 2) First-order PDEs: traffic flow equation, method of the characteristics, conservations laws, Burgers equation 3) Second-order PDEs: classification of second-order PDEs, the classical trinity (Diffusion, Wave and Laplace equations), direct extensions (nonlinear diffusion equation), applications to musical instruments 4) Further advanced topics in physical, life and social sciences: examples could include reaction-diffusion equation (wave propagation and pattern formation in biology), chemotaxis, swarming, population dynamics, financial markets and Black-Scholes equation, electrodynamics, fluid dynamics. 5) A short introduction to numerical methods.

MATH50004	Multivariable Calculus and differential Equations	Dr. A. Walton Professor M. Rasmussen	93.31%	Differential and integral calculus. Volume II, Richard Courant	1) Introduction to vector calculus: tensor notation and summation convention. 2) Differential operators: gradient, divergence and curl, operations with the gradient, Laplacian, scalar and vector fields; 3) Elements of integration: line, surface and volume integrals, Green's theorem, divergence theorem, Gauss' theorem, Stokes' theorem; 4) Curvilinear coordinates: implicit/inverse function theorems, line and volume elements, gradient, divergence, curl and Laplacian in curvilinear coordinates, changes of variables (jacobian);12 5) Calculus of variations: Derivation of Euler-Lagrange Equation; short forms of the equation; extension to constrained problems and to higher dimensions; applications including catenary, brachistochrone, geodesics in various geometries. Term 2: Part II - Ordinary Differential Equations: 1) Ordinary differential equations and initial value problems;

					2) Existence and uniqueness of solutions of ODEs: Picard iterates, metric spaces and normed vector spaces, Banach fixed point theorem, Picard-Lindelof theorem, maximal solution, general solutions and flows; 3) Linear systems: algebraic structure of the space of solutions, matrix exponential function, planar linear systems, Jordan normal form, exponential growth behavior, variation of constants formula; 4) Nonlinear systems: stability, invariant sets and limits sets, Lyapunov functions, Poincaré- Bendixson theorem.
MATH50003	Linear Algebra and Numerical Analysis	Professor M. Liebeck Dr. Sheehan Olver	88.06%	Linear Algebra, Serge lang	This module is composed of two parts: - Part I - Linear Algebra in Term 1; - Part II - Numerical Analysis in Term 2; An indicative list of sections and topics covered in both parts of the module is as follows: Part I - Linear Algebra (term 1; 20 lectures): 1) Direct sums and quotient spaces in vector spaces; invariance of these under a linear map

					and related matrices. Triangular form (over complex numbers) and Cayley - Hamilton. 2) Factorization of polynomials (over fields); minimal polynomial of a linear map. 3) Canonical forms: primary decomposition, cyclic decomposition, rational and Jordan canonical form (proofs not examinable). 4) Bilinear maps and forms; inner products. Examples. Gram - Schmidt again. Quadratic forms. Annihilators. Dual spaces. Part II - Numerical Analysis (term 2): 1) Brief review of Inner product spaces, Gram-Schmidt, Cauchy-Schwarz inequality. 2) Floating point arithmetic and stability of algorithms. 3) Numerical Linear Algebra: orthogonal matrices, positive definite matrices, Cholesky factorization. 4) Orthogonal Polynomials: three term recurrence relationship, Chebyshev polynomials. 5) Polynomial Interpolation: Lagrange and Chebyshev interpolation, existence and uniqueness, divided differences, error analysis.

					6) Numerical Integration and Differentiation. Approximation to ODEs. 7) Implementation of algorithms in e.g. Julia, Python or Matlab
MATH50006	Lebesgue Measure and Integration	Dr. P. F. Ridriguez	95.08%	Measures, Integrals and Martingales 2nd Edition, Rene Schilling	An indicative list of sections and topics is: Motivation: Drawbacks of the Riemann integral, Limits of functions, Length and area, The Fundamental Theorem of Calculus, Measures of sets in R^d, Measure Theory: Abstract measure theory: Motivation, basic definitions of measure spaces and measures. Lebesgue Measure in R^d: Volume of rectangles and cubes, The exterior measure, properties, Lebesgue measurable sets, countable additivity. Properties of the Lebesgue measure, Regularity, Invariance, σ-algebras and Borel sets, Non-measurable set. Measurable functions: Definitions and equivalent formulations of measurability, Sums and products, compositions, limits of measurable functions, “Almost everywhere” properties.

					Approximation by simple functions and step functions, Egoroff's and Lusin's theorems. Lebesgue integration: Definition using bounded functions on sets of finite measure, Riemann integrable functions are Lebesgue integrable, Integrable functions as a normed vector space, L_p(R^d), dense subsets, Completeness, Fatou's Lemma, monotone convergence theorem, uniform integrability, Vitali's Theorem, Fubini's and Tonelli's Theorems, statements and proofs. Differentiation and Integration: Differentiation of the Integral, statement of Lebesgue differentiation theorem, Differentiation of functions, Functions of bounded variation, properties, characterisation, Bounded variation implies differentiable a.e. Absolute continuity of measures, decomposition theorems by Jordan, Hahn and Lebesgue, Radon-Nikodym Theorem.

MATH50005	Groups and Rings	Professor A. A. Skorobogatov Professor T. Coates	92.88%	Rings, Fields and Groups by R.B.J.T. Al- lenby, second edition (1991).	An indicative list of sections and topics is: Groups: Further examples. Normal subgroups, quotient groups and the 1st isomorphism theorem. Finitely generated abelian groups (via Smith normal form). Group actions, orbit- stabiliser and simple applications. Rings: Definitions and examples (mainly commutative). Units and zero-divisors. Integral domains, Euclidean domains and unique factorisation. Ideals and quotient rings, first isomorphism theorem. Characteristic of an ID; construction of finite fields.
MATH50010	Probability for Statistics	Dr. C. Hallsworth	93.98%	Probability and random processes, by Grimmett, Geoffrey	An indicative list of sections and topics is: - Probability spaces - The Borel sigma algebra - Countable additivity and the continuity property - Univariate and multivariate random variables - Transformations of multivariate random variables - Modes of convergence of random variables - Laws of large numbers

					- Joint moment generating functions - Central limit theorem (including proof) - Random walks (1D) - Discrete time Markov chains with finite state space - Transition probabilities and matrices - Chapman-Kolmogorov equations - Expected hitting times and probabilities - Classification of states - Limiting and stationary distributions
MATH40005	Probability and Statistics	Professor A. Veraart Dr. D. Bodenham	91.28%	"A First Course in Probability", Sheldon Ross, Collier Macmillan, 1988	I - Interpretations of probability: limiting frequency; classical (symmetry between equally likely outcomes); subjective (degree of personal belief) II - Counting: multiplication principle; binomial coefficients; the inclusion-exclusion principle stars and bars arguments III - Formal probability: probability axioms; conditional probability; Bayes' theorem; independence IV - Random variables: mass and density functions; common discrete and continuous distributions; transformations of random variables; expectation and variance; probability and moment generating functions V - Multivariate random variables: joint mass and density functions; independence; covariance VI - Conditional distribution: conditional probability mass function; conditional density; conditional

					expectation; law of total expectation VII - Properties of random samples: a statistic and its sampling distribution; estimators, moments, maximum likelihood; exploratory data analysis VIII - Resampling methods: the bootstrap IX - Linear models: simple linear regression (continuous predictors); R^2; properties of residuals X - Hypothesis testing: Fisher's exact test; Student's t-test XI - Study design: observational vs experimental comparisons; confounding and bias Statistics: VII - Properties of random samples: a statistic and its sampling distribution; estimators, moments, maximum likelihood; exploratory data analysis VIII - Resampling methods: the bootstrap IX - Linear models: simple linear regression (continuous predictors); R^2; properties of residuals X - Hypothesis testing: Fisher's exact test; Student's t-test XI - Study design: observational vs experimental comparisons; confounding and bias
MATH40003	Linear Algebra and Groups	Dr. C. Kestner Professor D. Evans	83.05%	"Linear Algebra", R B J T Allenby, Edward Arnold, 1995, and A first course in abstract algebra, by Fraleigh, John B.	Linear Algebra: (i) Systems of linear equations: Equivalent systems; the augmented matrix; elementary row operations; Gaussian elimination. Examples and geometric interpretation (rotations and reflections in 2- and 3-dimensions). The theorem that a homogeneous system

					of linear equations with more unknowns than equations has a non-trivial solution. Brief discussion of fields. (ii) Matrix Algebra (over a field): Addition and multiplication of matrices. Matrices as linear transformations. Connection with solving linear equations. The inverse of a square matrix. Singular matrices and non-invertibility. Method for inverting a square matrix. (iii) Vector spaces (over a field): axioms and simple deductions from them; key examples (including function spaces); subspaces; linear independence and linear span; bases and dimension; dimension of subspaces; modular law. Applications and computations. (iv) Linear transformations: Examples; kernel and image; rank + nullity Matrix of a linear transformation with respect to given bases. Geometric examples (projections and rotations). Change of basis formula for linear transformations. (v) Determinants: review of 2×2 and 3×3 cases. Definition of general case by 1-st row expansion. Properties and equivalent ways of computing determinants. Special examples (including Vandermonde). det(AB) = det(A)det(B) and det(A) = det(A^T). Inverting matrices by using determinants. (vi) Eigenvalues and eigenvectors; Characteristic polynomial and invariance under change of basis; computations; diagonalisability and applications. Orthonormal bases and Gram- Schmidt. Diagonalisability of symmetric matrices over R and applications.

					Group theory: Groups: Axioms and simple deductions from them; subgroups, orders of elements. Examples (cyclic, symmetric groups, general linear groups). Cosets; Lagrange’s theorem and applications. Homomorphisms and isomorphisms (straightforward examples). Cycle structure, order and sign of permutations. Dihedral groups.
MATH50002	Analysis 2	Dr. D. Cheraghi Professor A. Laptev	95.75%	Rudin, Principles of Mathematical Analysis	An indicative list of sections and topics is: Term 1; 20 lectures: Higher dimensional derivatives: Definition of higher dimensional derivative, chain rule. Directional derivatives, partial derivatives, Df(p) in terms of partial derivatives, Higher derivatives, higher dimensional Taylor’s theorem, Symmetry of mixed partials (statement of results). Inverse function theorem, implicit function theorem. Metric spaces: Definition, examples. Topologically equivalent metrics, isometries, Lipschitz maps, Open sets, bounded sets, examples, unions, intersections, Continuity in terms of open

					sets, Closed sets, closure, limit points, Separable metric spaces, Topological spaces. Compact spaces: Definition in terms of open covers, Basic features, existence of convergent sub-sequences, Continuous maps and compact sets, Sequential compactness. Completeness: Definition, examples, Point-wise convergence and uniform convergence in function spaces, Incompleteness of C(X), Continuity of the integration on function spaces, Arzela-Ascoli, Fixed point theorem. Connectedness: Definition, examples, Continuous image of a connected set. Term 2: 20 lectures, Complex Analysis: Holomorphic Functions: Definition using derivative, Cauchy-Riemann equations, Polynomials, Power series,9 Rational functions, Moebius transformations, Cauchy's Integral Formula: Complex integration along curves, Goursat's theorem, Local existence of primitives and Cauchy's theorem in a disc, Evaluation of some integrals,

					Homotopies and simply connected domains, Cauchy's integral formulas. Applications of Cauchy's integral formula: Morera's theorem, Sequences of holomorphic functions, Holomorphic functions defined in terms of integrals, Schwarz reflection principle. Meromorphic Functions: Zeros and poles. Laurent series. The residue formula, Singularities and meromorphic functions, The argument principle and applications, The complex logarithm. Harmonic functions: Definition, and basic properties, Maximum modulus principle. Conformal Mappings: Definitions, Preservation of Angles, Statement of the Riemann mapping theorem.
MATH40002	Analysis 1	Dr. A. Chandra Dr. S. Sivek	96.26%	"Mathematical Analysis", K G Binmore, Cambridge University Press, 1977	I Real Numbers: The archimedean property, and density of Q in R; The completeness axiom; Sup and Inf, and basic properties; Decimal Expansions; Countability and uncountability II Real and complex sequences: Convergence and Divergence; the sandwich test; Sub- sequences, monotonic sequences, [limsup and liminf,] Bolzano-

					Weierstrass Theorem, Cauchy sequences and the general principle of convergence. III - Real and complex series: Convergent and absolutely convergent series; Comparison test for non-negative series and for absolutely convergent series; Alternating test series; Rearranging absolutely convergent series Radius of convergence of power series; Exponential series. IV - Continuity of real and complex functions: Left and right limits and continuity for real and complex functions; Sequential criterium for continuity; uniform continuity; Compact sets and extrema of real valued continuous functions; Inverse function theorem for strictly monotonic real functions on an interval. V - Differentiability: Definitions and examples; Left and right derivative, properties of derivatives. Higher derivatives, convexity; Differentiation of series. VI - Integrability: Integral for step functions; Definition of Riemann-Darboux integral and examples of integrable/non-integrable functions; Elementary properties; The Mean Value Theorem for Integrals; The Fundamental theorem of Calculus; Integration by parts, substitution.

MATH40004	Calculus and Applications	Professor D. Papageorgiou	89.14%	Calculus, by James Stuart	I - Differentiation: First principles, differentiability; Logarithmic and implicit differentiation; Higher derivatives; Leibniz's formula; Stationary points and points of inflexion. II - Graphs: Curve sketching; Parametric representation; Polar coordinates; Complex graphs. III - Series: Convergence of Infinite Series; Maclaurin and Taylor expansions; The Mean Value Theorem; Taylor's Theorem with remainder; Convergence of Infinite Power Series; L'Hopital's rule; Complex power series. IV - Integration: The Riemann integral; Fundamental theorem of calculus; Indefinite integrals; Integration by parts and substitution; Partial fractions; Differentiating under the integral; Improper integrals;Integrals over curves, areas and volumes with examples including moments of inertia. V - Fourier Series: orthonormal systems; periodic functions; even and odd functions; full- range and half-range series; the Gibbs phenomenon; Parseval's theorem; integration and differentiation of Fourier series; exponential form. VI - Fourier Transforms: Exponential, cosine and sine transforms; Elementary properties; Convolution theorem; Energy theorem. VII - Ordinary Differential Equations: Introduction to ODEs: definitions and notations; Solutions for 1st and some 2nd order ODEs, linear ODEs; Separable, homogeneous and linear equations; Special cases; Linear higher order equations with constant coefficients; Systems of constant-coefficient linear ODEs; Qualitative Analysis of linear ODEs: Phase plane

					Analysis, stability of systems; Qualitative Analysis of nonlinear ODEs: Bifurcation Analysis. Including numerous examples from Newtonian dynamics such as motion point particle in an external potential and oscillatory motion. VIII - Introduction to Multivariable Calculus: General properties of functions of several variables; Partial derivatives and total derivatives; Second order derivatives and statement of condition for equality of mixed partial derivatives; Taylor expansions; Chain rule, change of variables, including planar polar coordinates.
MATH40006	Introduction to Computation	Dr. P. Ramsden	91.6%		1) Introduction The relationship between computing and mathematics - Programming languages - Python: versions, distributions and interfaces - Jupyter notebooks and markdown - Git and Github - Calculations - Variables and assignment - The math and cmath modules 2) Core data types and algorithms Introduction to functions - Native data types in Python - Simple native data structures in Python - Iteration, branching and recursion - Comprehensions and filtering - Iterable objects in loops and comprehensions - Further functions - Algorithms and efficiency 3) Modules, further data structures and files The NumPy module - The matplotlib module and the pyplot submodule - Data analysis using NumPy and

					matplotlib - User-defined modules - Further data structures - File I/O
MATH40007	An Introduction to Applied Mathematics	Professor D. Crowdy	91.5%		1) Structures in equilibrium: trusses (example of a directed graph), force balance and equilibrium 2) Edge-node incidence matrix, constitutive matrix and stiffness matrix 3) Equilibrium equations with external forces 4) Applied linear algebra for solving the equilibrium equations using Gauss elimination, eigenvalues/eigenvectors 5) Equilibrium as energy minimization: connection with least-square problems 6) Description of analogous systems: e.g. Kirchhoff's equations, heat transfer and other transport processes 7) Continuous limit in the one-dimensional case 8) Introduction to eigenfunctions (with the example of a Sturm-Liouville system) 9) Minimum principles and calculus of variations in 1D 10) Continuous limit in the two-dimensional case 11) Stokes/divergence theorems (as motivated by the physical problems) 12) Laplace's equation and 2D potential theory 13) Characteristics coordinates of Laplace's equation and complex variables 14) Introduction to analytic functions 15) Conformal mapping and Fourier series 16) Hall effect in transport theory

MATH40001	Introduction to University Mathematics	Professor K. Buzzard	Pass		Sets and Logic – Notation and basic results for sets: ∈, ⊆ etc; de Morgan. – Propositions are true-false statements. – Logical notation: ∀, ∧, ¬, =⇒ etc – Truth tables for basic logical connectives. – Basic proof strategies: direct, induction, contradiction, contrapositive. – Basic examples of proofs. – Negation of ”for all x, there exists y such that ...” etc. Integers – Induction – Division with remainder; Euclidean algorithm – Prime numbers, infinitude of primes – Fundamental Theorem of Arithmetic – Modular arithmetic – Fermat’s Little Theorem Functions and Equivalence relations – Functions (injectivity, surjectivity and bijectivity); composition; existence of an inverse. – Equivalence relations and equivalence classes Real Numbers – Theory of inequalities, built from an axiomatic viewpoint. – Axioms and basic proofs in the theory of ordered fields. Vectors and Geometry

					– Vector algebra: Manipulation (addition, substraction) of vectors in 2D and 3D spaces – Geometry of vectors: Definition of the dot-product, Cauchy-Schwarz inequality and Triangle inequality, angles between vectors. – Systems of coordinates (Cartesian, Polar, Cylindrical and Spherical) – Definition of the cross-product (including as a determinant) – Right-handedness, scalar triple product, vector triple product – 2D and 3D elementary geometry: equations of lines and planes – Relations with distances, areas and volumes.